

# COMMENT ON “A VIRTUAL SENSING METHOD FOR TONAL ANVC SYSTEMS” [JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL, VOL. 124]

**Cornelis D. Petersen\***

Active Noise and Vibration Control Group  
School of Mechanical Engineering  
The University of Adelaide  
South Australia, 5005  
Email: cornelis.petersen@adelaide.edu.au

**Rohin Wood  
Ben S. Cazzolato  
Anthony C. Zander**

Active Noise and Vibration Control Group  
School of Mechanical Engineering  
The University of Adelaide  
South Australia, 5005

## ABSTRACT

*In a paper by Tran and Southward [1], a virtual sensing method for tonal active noise and vibration control systems is proposed. The aim of the proposed method is to obtain accurate estimates of the virtual outputs of the dynamic system under consideration. For this purpose, a hybrid adaptive feedforward observer is designed based on an observable state-space representation of the dynamic system. In this paper, it is shown that if the number of physical sensors used in the proposed method is less than the state-space system order, the observer can converge to infinitely many solutions for which the state reconstruction errors are not equal to zero. Since accurate state estimates are required to obtain accurate estimates of the virtual sensor outputs, the suggested hybrid adaptive feedforward observer is only suitable for rejecting non-stationary disturbances at the physical sensor outputs, and not for virtual sensing purposes.*

## INTRODUCTION

In a paper by Tran and Southward [1], a virtual sensing method for tonal active noise and vibration control systems is proposed. The proposed method uses a hybrid adaptive feedforward observer that is designed using an observable state-space representation of the dynamic system under consideration. This observer aims to provide accurate estimates of the true states of the dynamic system, such that accurate estimates of the virtual outputs can be obtained by direct mapping from the observer

states. In this paper, it is shown that if the number of physical sensors used in the observer is smaller than the state-space system order, which is almost always the case, it is not always guaranteed that the state reconstruction errors are equal to zero after convergence of the hybrid adaptive feedforward observer. This means that the proposed method is not suitable for virtual sensing purposes.

## PROPOSED METHOD

In this section, the virtual sensing method proposed by Tran and Southward is introduced. For a more detailed description, one is referred to the original paper [1]. A block diagram of the problem under consideration is shown in Fig. 1. A state-space system description of the plant in this figure is given by

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{w} \\ \mathbf{y}_p &= \mathbf{C}_p\mathbf{x} + \mathbf{D}_p\mathbf{u} \\ \mathbf{y}_v &= \mathbf{C}_v\mathbf{x} + \mathbf{D}_v\mathbf{u},\end{aligned}\tag{1}$$

with  $\mathbf{x}$  the  $N$  states of the system,  $N$  the system order,  $\mathbf{y}_p$  the outputs at the  $M$  physical sensors,  $\mathbf{y}_v$  the outputs at the  $M_v$  virtual sensors,  $\mathbf{u}$  the  $L$  control inputs, and  $\mathbf{w}$  a vector of length  $N$  that contains the unknown external disturbances. It is assumed that no a priori knowledge is available of how the external disturbances  $\mathbf{w}$  affect the states of the plant [1]. However, as indicated in Fig. 1 by the dashed line, a feedforward reference signal which is correlated to the external disturbances  $\mathbf{w}$  is assumed to be available.

---

\*Address all correspondence to this author.

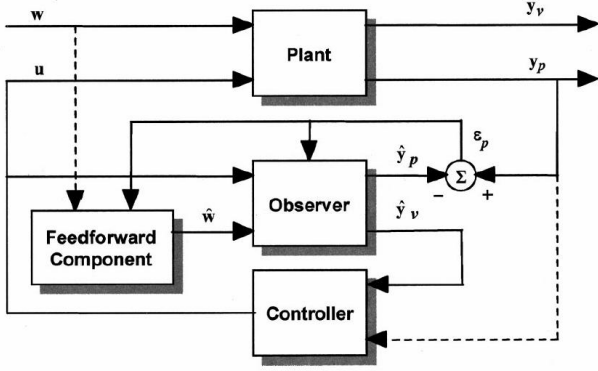


Figure 1. Block diagram of the hybrid adaptive feedforward observer [1].

First, by initially assuming that there is no external disturbance input such that  $\mathbf{w} = \mathbf{0}$  in Eq. (1), a conventional observer is designed [2]. Next, the conventional observer is augmented with an adaptive feedforward component that aims to provide estimates  $\hat{\mathbf{w}}$  of the external disturbances  $\mathbf{w}$  in Eq. (1). A state-space description of the complete hybrid adaptive feedforward observer is given by [1]

$$\begin{aligned} \dot{\hat{\mathbf{x}}} &= \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}\mathbf{u} + \mathbf{F}\boldsymbol{\epsilon}_p + \dot{\hat{\mathbf{w}}} \\ \hat{\mathbf{y}}_p &= \mathbf{C}_p\hat{\mathbf{x}} + \mathbf{D}_p\mathbf{u} \\ \hat{\mathbf{y}}_v &= \mathbf{C}_v\hat{\mathbf{x}} + \mathbf{D}_v\mathbf{u} \end{aligned}, \quad (2)$$

where  $\boldsymbol{\epsilon}_p = \mathbf{y}_p - \hat{\mathbf{y}}_p$  is a vector of length  $M$  that contains the physical sensor output errors. The aim of the observer in Eq. (2) is to obtain accurate estimates  $\hat{\mathbf{x}}$  of the true plant states. These estimates, together with the known control inputs  $\mathbf{u}$ , are then used to obtain an estimate  $\hat{\mathbf{y}}_v$  of the virtual sensor output. In order to obtain accurate estimates of the virtual outputs  $\mathbf{y}_v$ , such that the virtual sensor output errors  $\boldsymbol{\epsilon}_v = \mathbf{y}_v - \hat{\mathbf{y}}_v$  are small, the observer needs to provide accurate estimates of the true states  $\mathbf{x}$  of the plant. If the state reconstruction error is defined to be  $\boldsymbol{\rho} = \mathbf{x} - \hat{\mathbf{x}}$ , the estimator error equation can be written as [1]

$$\begin{aligned} \dot{\boldsymbol{\rho}} &= [\mathbf{A} - \mathbf{F}\mathbf{C}_p]\boldsymbol{\rho} + \mathbf{w} - \dot{\hat{\mathbf{w}}} \\ \boldsymbol{\epsilon}_p &= \mathbf{C}_p\boldsymbol{\rho} \end{aligned}. \quad (3)$$

Eq. (3) shows that the state reconstruction error  $\boldsymbol{\rho}$  can be reduced to zero if  $\dot{\hat{\mathbf{w}}}$  approaches  $\mathbf{w}$  [1]. It is then suggested by Tran and Southward that this objective can be achieved by minimizing the physical sensor output errors  $\boldsymbol{\epsilon}_p$  using an LMS-based adaptive algorithm that minimizes the cost function [1]

$$J = \frac{1}{2} \boldsymbol{\epsilon}_p^T \boldsymbol{\epsilon}_p. \quad (4)$$

Although the suggested adaptive observer indeed minimizes this cost function to zero such that  $\boldsymbol{\epsilon}_p = \mathbf{0}$ , it is proven in the next section that it will not always reduce the state reconstruction error  $\boldsymbol{\rho}$  to zero as suggested by Tran and Southward. The numerical results presented by Tran and Southward in Figs. 6 and 7 already illustrate this [1]. The results depicted in Fig. 6 of their paper indicate that the physical sensor output errors have indeed been minimized such that  $\boldsymbol{\epsilon}_p = \mathbf{0}$  after convergence of the adaptive algorithm. If the observer would work as suggested, the virtual sensor output errors would be minimized to  $\boldsymbol{\epsilon}_v = \mathbf{0}$  as well, since the state reconstruction errors would then be  $\boldsymbol{\rho} = \mathbf{0}$  (according to Tran and Southward). Comparing Figs. 6 and 7 in their paper, it can be seen that this is clearly not the case because the virtual sensor output errors  $\boldsymbol{\epsilon}_v$  have not converged to zero in Fig. 7. This indicates that, although the physical sensor output errors  $\boldsymbol{\epsilon}_p$  are indeed minimized to zero, accurate estimates  $\hat{\mathbf{x}}$  of the plant states are not obtained, and the state reconstruction errors  $\boldsymbol{\rho}$  are not equal to zero after convergence of the adaptive algorithm. In the next section, it is shown that the adaptive algorithm can converge to infinitely many solutions  $\hat{\mathbf{w}}$  that drive the cost function in Eq. (4) to zero, but that do not necessarily set the state reconstruction error equal to the desired solution  $\boldsymbol{\rho} = \mathbf{0}$ .

## PROOF

By substituting  $\boldsymbol{\epsilon}_p = \mathbf{C}_p\boldsymbol{\rho}$  into the cost function  $J$  in Eq. (4), this cost function can also be written as

$$J = \frac{1}{2} \boldsymbol{\rho}^T \mathbf{C}_p^T \mathbf{C}_p \boldsymbol{\rho}. \quad (5)$$

The LMS-based algorithm adjusts the estimate of  $\hat{\mathbf{w}}$  such that this cost function is minimized. In the case that the number of physical sensors  $M$  is less than the number of states  $N$ , which is almost always the case, the matrix  $\mathbf{C}_p$  will have rank  $\leq M$ . Due to the rank-nullity theorem [3], the matrix  $\mathbf{C}_p$  will therefore have a nullspace  $\mathcal{N}(\mathbf{C}_p)$  of dimension  $\geq N - M$ . Thus, the non-negative cost function in Eq. (5) is minimized for any  $\boldsymbol{\rho} \in \mathcal{N}(\mathbf{C}_p)$ . An equivalent observation is that the matrix  $\mathbf{C}_p^T \mathbf{C}_p$  in Eq. (5) will be positive semi-definite if  $M < N$ , but not positive definite as required to ensure that  $\boldsymbol{\rho}$  converges to zero. It follows that the LMS-based algorithm will only ensure  $\boldsymbol{\rho}$  converges to the subspace  $\mathcal{N}(\mathbf{C}_p)$ , and not to the desired value  $\boldsymbol{\rho} = \mathbf{0}$  as suggested by Tran and Southward [1].

It is now shown that once  $\boldsymbol{\rho}$  has converged to  $\mathcal{N}(\mathbf{C}_p)$ , the structure of the hybrid observer will allow it to stay there. Suppose that a basis  $S$  for  $\mathcal{N}(\mathbf{C}_p)$  is given by [3]

$$S = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n\}, \quad (6)$$

with  $n = \text{nullity}(\mathbf{C}_p)$ . The values of  $\boldsymbol{\rho} \in \mathcal{N}(\mathbf{C}_p)$  that minimize

the cost function in Eq. (5) can now be written as

$$\begin{aligned}\boldsymbol{\rho} &= \alpha_1 \mathbf{v}_1 + \alpha_2 \mathbf{v}_2 + \dots + \alpha_n \mathbf{v}_n \\ &= \mathbf{V}\boldsymbol{\alpha},\end{aligned}\quad (7)$$

where the columns of the  $N \times n$  matrix  $\mathbf{V}$  consist of the basis vectors  $\mathbf{v}_i$  defined in Eq. (6), and where  $\boldsymbol{\alpha}$  is a column vector of length  $n$  with arbitrary real-valued elements  $\alpha_i$ . Substituting Eq. (7) into the estimator error equation defined in Eq. (3) gives

$$\begin{aligned}\dot{\boldsymbol{\rho}} &= \mathbf{A}\mathbf{V}\boldsymbol{\alpha} + \mathbf{w} - \hat{\mathbf{w}}, \\ \boldsymbol{\varepsilon}_p &= \mathbf{0},\end{aligned}\quad (8)$$

since  $\mathbf{C}_p \mathbf{V}\boldsymbol{\alpha} = \mathbf{0}$  as  $\mathbf{V}\boldsymbol{\alpha}$  defines any arbitrary vector in  $\mathcal{N}(\mathbf{C}_p)$ . For  $\boldsymbol{\rho}$  to stay in  $\mathcal{N}(\mathbf{C}_p)$ , such that the cost function in Eq. (4) has been minimized with  $\boldsymbol{\varepsilon}_p = \mathbf{0}$ , it can be easily shown that the time-derivative  $\dot{\boldsymbol{\rho}}$  of the state reconstruction error must be in  $\mathcal{N}(\mathbf{C}_p)$  as well. This means that  $\dot{\boldsymbol{\rho}}$  can be expressed as a linear combination of the vectors  $\mathbf{v}_i$  defined in Eq. (6) as

$$\begin{aligned}\dot{\boldsymbol{\rho}} &= \alpha_1 \mathbf{v}_1 + \beta_2 \mathbf{v}_2 + \dots + \beta_n \mathbf{v}_n \\ &= \mathbf{V}\boldsymbol{\beta}.\end{aligned}\quad (9)$$

with  $\boldsymbol{\beta}$  a column vector of length  $n$  with arbitrary real-valued elements  $\beta_i$ . Substituting Eq. (9) into Eq. (8) gives

$$\begin{aligned}\mathbf{V}\boldsymbol{\beta} &= \mathbf{A}\mathbf{V}\boldsymbol{\alpha} + \mathbf{w} - \hat{\mathbf{w}}, \\ \boldsymbol{\varepsilon}_p &= \mathbf{0},\end{aligned}\quad (10)$$

Eq. (10) shows that, once the adaptive algorithm reaches a solution such that  $\hat{\mathbf{w}}$  is given by

$$\hat{\mathbf{w}} = \mathbf{w} + \mathbf{A}\mathbf{V}\boldsymbol{\alpha} - \mathbf{V}\boldsymbol{\beta},\quad (11)$$

the cost function in Eq. (5) has been minimized as both  $\boldsymbol{\rho} \in \mathcal{N}(\mathbf{C}_p)$  and  $\dot{\boldsymbol{\rho}} \in \mathcal{N}(\mathbf{C}_p)$ , which means that the state reconstruction error stays equal to  $\boldsymbol{\varepsilon}_p = \mathbf{0}$ . As the elements of  $\boldsymbol{\alpha}$  and  $\boldsymbol{\beta}$  can be set to any arbitrary real value in Eq. (11), the adaptive algorithm can converge to infinitely many solutions  $\hat{\mathbf{w}}$  that are not equal to  $\mathbf{w}$ . Once the adaptive algorithm has converged to one of these solutions, the state reconstruction error  $\boldsymbol{\rho}$  enters and stays in  $\mathcal{N}(\mathbf{C}_p)$ . In other words, the adaptive algorithm can converge to infinitely many solutions  $\hat{\mathbf{w}}$  that minimize the cost function in Eq. (4), but that do not necessarily set the state reconstruction error equal to the desired solution  $\boldsymbol{\rho} = \mathbf{0}$ . In order to verify this, numerical simulations of the method proposed by Tran and Southward were performed for an acoustic duct. An analysis of

these simulations indicated that the obtained numerical results were similar to the ones presented by Tran and Southward [1]. Moreover, it was found that the state reconstruction error indeed converged into  $\mathcal{N}(\mathbf{C}_p)$ , and not to zero as suggested by Tran and Southward.

In conclusion, if the number of physical sensors  $M$  is less than the system order  $N$ , the method suggested by Tran and Southward indeed minimizes the physical sensor output errors such that  $\boldsymbol{\varepsilon}_p = \mathbf{0}$  after convergence, but it does not guarantee that the state reconstruction errors converge to the desired values  $\boldsymbol{\rho} = \mathbf{0}$ . Since accurate state estimates are required to obtain accurate estimates of the virtual sensor outputs  $\mathbf{y}_v$ , the suggested hybrid adaptive feedforward observer is only suitable for rejecting non-stationary disturbances at the physical sensor outputs, and not for virtual sensing purposes.

## CONCLUSIONS

In this paper, the virtual sensing method proposed by Tran and Southward was analysed [1]. It was shown that if the number of physical sensors used in the algorithm is smaller than the state-space system order, the proposed hybrid adaptive feedforward observer does minimize the physical sensor output errors such that the physical output errors are equal to zero after convergence, but it does not guarantee that the state reconstruction errors converge to zero as well. Since accurate state estimates are required to obtain accurate estimates of the virtual sensor outputs, the proposed method is only suitable for rejecting non-stationary disturbances at the physical sensor outputs, and not for virtual sensing purposes.

## ACKNOWLEDGMENT

The author gratefully acknowledges The University of Adelaide for providing an ASI scholarship, and the Australian Research Council for supporting this research.

## REFERENCES

- [1] Tran, C. M., and Southward, S. C., 2002. "A virtual sensing method for tonal anvc systems". *Journal of Dynamic Systems, Measurements, and Control*, **124**(1), March, pp. 35–40.
- [2] Franklin, G. F., Powell, J. D., and Enami-Naeini, A., 1999. *Feedback Control of Dynamic Systems*, 3<sup>rd</sup> ed. Addison-Wesley Publishing Company.
- [3] Anton, H., and Torres, C., 1991. *Elementary Linear Algebra*, 6<sup>th</sup> ed. John Wiley & Sons, Inc.